Resolving our erroneous interpretation of the Galilean Transformation

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Abstract: Our interpretation of the Galilean Transformation has been incorrect. This error was carried forward by Einstein, who incorporated it into his Theory of Relativity. This resulted in the derivation of a theorem stating "time in an inertial frame, which is in motion at a constant velocity with respect to an observer in a static reference frame, lags behind the time in the static system." The Twin Paradox is a result of this theorem. Our interpretation of the Galilean Transformation, which has been erroneous until now, is herein corrected. The present paper provides a correct interpretation of the Theory of Relativity, and the Twin Paradox is resolved.

Key words: Galilean Transformation; Lorenz Transformation; Relativity; Time Dilation; Twin Paradox

I. The Twin Paradox

Einstein provided the following explanation in his Special Theory of Relativity presented in 1905:

"If at the points A and B of a stationary system there are stationary clocks which, viewed in the stationary system, are synchronous; and if the clock at A is moved with the velocity v along the line AB to B, then on its arrival at B the two clocks no longer synchronize, but the clock moved from A to B lags behind the other which has remained at B by $\frac{1}{2}tv^2/c^2$ (up to magnitudes of fourth and higher order), t being the time occupied in the journey from A to B".[1]

According to the Principle of Relativity, it would be acceptable to reverse the explanation provided above, namely that "Observer A remained at standstill while Observer B traveled to the location of Observer A." In such cases, the clock of Observer B would indicate a time that lags behind the time indicated by the clock of Observer A. This is contrary to the assertion described earlier, namely that the "time indicated by the clock of Observer A lags behind that indicated by the clock of Observer B."

The fact that the assertions regarding time mutually contradict each other, as described above, has been known as the Twin Paradox in the Special Theory of Relativity. Physicists have even provided numerous experimental data as corroborative evidence for the time delay according to Einstein's Theory of Relativity. In spite of such efforts, however, this problem has not been resolved thus far.[2, 3, 4, 5]

II. Correct interpretation of the Galilean Transformation

Let us for the moment assume that there are two inertial reference frames in which Observer A and Observer B exist, as shown in Fig. 1. In this case, Observer A observes that the system in which Observer B exists is moving away at a constant velocity v while the x axis and the X axis remain parallel.

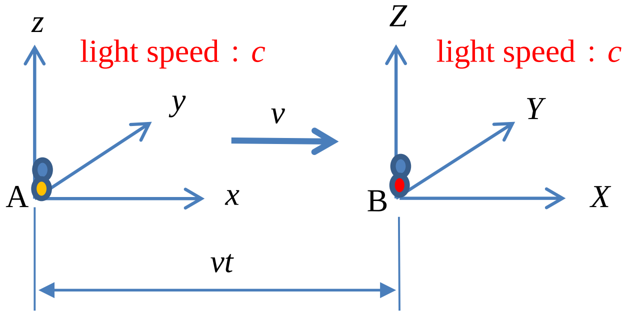

Figure 1. Relationship between two inertial systems and the Galilean Transformation

According to the Principle of Relativity, however, Observer B would observe the system in which Observer A is moving away at the constant velocity $-v$. The dynamics observed by Observers A and B in both cases are the dynamics that can be observed from a stationary system.

Observer A can construct a new reference frame that moves in parallel alongside Observer B by implementing an appropriate coordinate transformation. If Observer A transfers to such a new moving reference frame, Observer B will be observed as being stationary. The dynamic phenomenon observed by Observer A from such a new moving reference frame is referred to as relativistic dynamics.

The Galilean Transformation had been implemented in the past to discuss relativistic dynamics from the viewpoint of Observer A. The Galilean Transformation is generally given in the following manner:

$X = x - vt$ (1)

$Y = y$ (2)

$Z = z$ (3)

$T = t$ (4)

where (x, y, z) and t represent the spatial coordinates and time, respectively, of Observer A. Furthermore, (X, Y, Z) and T represent the spatial coordinates and time, respectively, of Observer B.

The Galilean Transformation implemented in this instance indicates that Observer A is transformed onto the system of Observer B through the transformation, as shown by Eqs. (1-4). This means that the perspective of Observer A is replaced by the perspective of Observer B.

These results in Observer A, who has undergone the Galilean Transformation, observing the same dynamics as those observed by Observer B in his own system. Furthermore, the velocity of light observed by Observer A (i.e., Observer B) appears to be isotropic and has a constant value c, according to the Principle of Relativity. This means that Observer A is not observing relativistic dynamics in the system of Observer B at all. This is our conventional error in interpreting the Galilean Transformation.

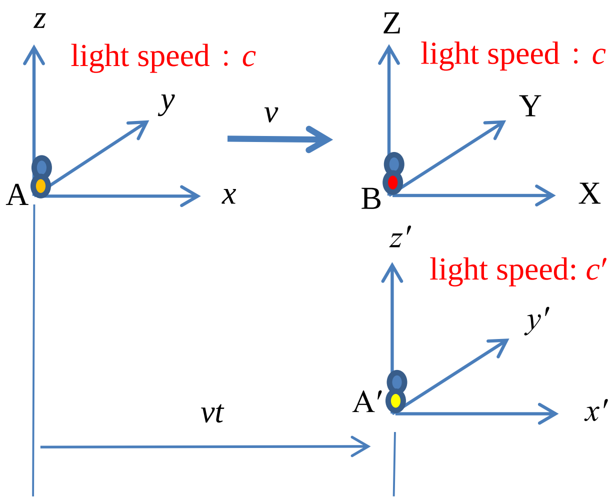

Figure 2. Relationship of two inertial systems and the correct coordinate transformation

The purpose of Observer A had essentially been to observe relativistic dynamics through the Galilean Transformation. The Galilean Transformation, when applied correctly, must be described as follows:

$x' = x - vt$ (5)

$y' = y$ (6)

$z' = z$ (7)

$t' = t$ (8)

where (x', y', z'), and t' are spatial coordinates and time, respectively, of a new moving reference frame set by Observer A. This is referred to as the system of Observer A′ as shown in Fig. 2.

The Observer B is observed to be at rest relative to Observer A (i.e., Observer A′), whose observation location has been transferred to the new moving reference frame through the transformation shown by Eqs. (5–8). The velocity of light, which is observed as being isotropic and as propagating at a constant velocity *c* in the original stationary system of Observer A or the system of the Observer B, would be observed differently by Observer A′ in the new moving reference frame, depending on the direction.

For instance, the velocity of light propagating in the direction of the x' axis would be observed by Observer A′ as c-v or c+v. Furthermore, the velocity of light propagating in the direction of the y' axis would be observed as c/γ, where the coefficient of γ is $1/\sqrt{1-v^2/c^2}$.

III. Correct interpretation of the Special Theory of Relativity

Does a coordinate transformation exist in which the velocity of light observed by Observer A, whose viewpoint has been transferred to a new moving reference frame through a coordinate transformation, is isotropic and is a constant value *c*? The Lorentz Transformation is the answer to this question. The Lorentz Transformation is correctly given by the following equations:[6]

$x' = \gamma(x - vt)$ (9)

$y' = y$ (10)

$z' = z$ (11)

$t' = \gamma\left(t - \frac{vx}{c^2}\right)$ (12)

The new moving reference frame set by Observer A to observe relative mechanics through the transformation of Eqs. (9–12) is hereinafter referred to as the system of Observer A″.

The velocity of light observed by Observer A (i.e., Observer A″), whose perspective has been transferred to the new moving reference frame through the Lorentz Transformation shown in Eqs. (9–12), is observed as isotropic and as having a constant value *c*. Observer B, furthermore, is observed as being stationary from Observer A″, who exists in the moving system.

We should here Note that the ticking of time at precisely identical rates in the system of Observer B (T) and in the stationary system of Observer A (t) at all times is an essential condition for the preservation of symmetry between the two systems, the following relation is given:

$T = t$ (13)

The validity of equation (13) is guaranteed by the principle of relativity.

Einstein[1] provided the following coordinate transformation, contrary to Eqs. (9‑13):

$X = \gamma(x - vt)$ (14)

$Y = y$ (15)

$Z = z$ (16)

$T = \gamma\left(t - \frac{vx}{c^2}\right)$ (17)

The coordinates after transformation of Observer A are set as the coordinates of the system of Observer B, as shown on the left-hand sides of these equations. Our conventional error in the interpretation of the Galilean Transformation was carried forward in this manner by Einstein in his Special Theory of Relativity. This resulted in Einstein deriving the theorem for the dilation of time, as shown below:

$T = \sqrt{1 - \frac{v^2}{c^2}}\, t$ (18)

As a result, the Twin Paradox was born.

The relational expression in Eq. (18) derived by Einstein should have been correctly given as follows using Eqs. (9‑13):

$t' = \sqrt{1 - \frac{v^2}{c^2}}\, t$ (19)

This means that there is no room for deriving the Twin Paradox.

This conclusion may appear to be on a first glance to be contrary to the experimental results obtained by B. Rossi[2], which are considered to be physical observation results that support the lagging of time. The extension of the lifetime of cosmic muons given by Rossi is described as an effect of an increase in the relativistic inertial mass (and/or the relativistic energy) due to the relative velocity through the transformation shown in Eqs. (9–13) according to Observer A, who is considering the relativistic dynamics.

The relativistic inertial mass of muons as observed by Observer A on Earth is given in the following manner:

$m = \gamma m_0$ (20)

where m represents the relativistic inertial mass and m_0 represents the static inertial mass.

This means that the assertion of Observer A on Earth, who is observing the relativistic inertial mass of muons that impact Earth from space, is described as follows:

"The time required for the relativistic inertial mass m of a muon to decrease to 1/2 of its static inertial mass, m_0, was exactly γ times the half-life indicated by the static inertial mass m_0."

Meanwhile, one might ask: why does the body of experimental results on the time dilation amassed by the physics community thus far agree so well with Einstein's theoretical predictions? This question may be easily answered as follows.[7]

The values of the times and lengths reported to Observer B in the moving system by Observer A (i.e., observer A″), who is effectively at rest with respect to an observer in the moving system due to the coordinate transformation, are given by the primed symbols on the left hand side of equations (9-13). Additionally, data on times and lengths obtained within the moving system by Observer A″ at rest with respect to the moving system of Observer B are also expressed entirely in terms of the primed physical quantities; using equations (9-13) these may be communicated to the stationary system. Consequently, time from the stationary system will appear dilated when measured by Observer B in the moving system.

For example, suppose that light emitted from the stationary system requires 20 seconds to arrive at the moving system. This 20-second interval is the length of time measured by an observer in the moving system; however, an observer making measurements from the stationary system will observe a precisely identical travel time of 20 seconds. This is the significance of equation (13).

On the other hand, the timing information delivered to an observer in the moving system by light that arrives at the moving system from the stationary system is dilated relative to the 20-second interval—perhaps to 10 seconds, for example. This is the significance of equation (19).

In contrast to this sort of interpretation, the timing information carried by light propagating to a moving system from a stationary system is 20 seconds, identical to the observational data measured by the observer in the stationary system; the fact that the observer in the moving system measures a value of 10 seconds is the significance of Einstein's equation (18).

In contrast to Einstein's assumptions, the timing information delivered to the moving system by light arriving from the stationary system is in fact dilated to 10 seconds. On the other hand, the time elapsed within both the moving system and the stationary system during this interval was 20 seconds. During this interval both clocks emitted 20 pulses; therefore, the frequencies generated by the clocks in the moving and stationary systems are equal to each other and have a value of 1 pulse/second.

In contrast, if the moving system's clock pulses are measured based on the dilated time of 10 seconds, we count 2 pulses per second, and the period obtained by inverting this number is 1/2 seconds per pulse. Thus, our measurements suggest that the time displayed by clocks in the moving system (for example, clocks aboard satellites of GPS) is dilated. The time dilation demonstrated by the physics community thus far has mistakenly interpreted this phenomenon as a dilation of time itself in the moving system.

Although the fact that time in the stationary system measured by observers in the moving system appears to be dilated was noted by L. Essen[5], he did not carry the argument far enough to rectify Einstein's theory of relativity.

Finally, we here conclude that the correct Galilean Transformation is given by Eqs. (5-8), under the assumption $v^2/c^2 << 1$ upon Eqs. (9-12).

IV. Conclusion

The Twin Paradox derived from Einstein's Special Theory of Relativity has finally been unraveled. The key to the solution was not in the logic developed after implementing the Lorentz Transformation, as assumed in the past, but instead it was in its predecessor, the Galilean Transformation, which was beyond a shadow of doubt for us as we implemented it in the past. Our understanding that led us to discuss relativistic dynamics using the Galilean Transformation has been wrong. This error ended up being carried forward by Einstein to be incorporated into his Special Theory of Relativity. The discovery that our conventional interpretation of the Galilean Transformation was wrong can certainly be considered an "Egg of Columbus." By correcting the Galilean Transformation, the Lorentz Transformation implemented by Einstein was also corrected, and the Twin Paradox was resolved at that instant.

[1] A. Einstein, The principle of relativity, Dover Publication, Inc., New York (1923).
[2] B. Rossi and D. B. Hall, Variation of the rate of decay of mesotrons with momentum, Phys, Rev. 59, 3, 223-228 (1941).
[3] J. C. Hafele and R. E. Keating, Around-the-world atomic clocks, Science 177, 166, and 168 (1972).
[4] A. G. Kelly, Hafele and Keating tests: Did they prove anything? PHYSICS ESSAYS,

13 (4), pp.616-621 (2000).
[5] L. Essen, The Special theory of relativity: a critical analysis, Oxford Science Research Paper 5, pp.1-27 (1971).
[6] E. Nakaza, A key in the Galilean transformation to solve the twin paradox in Einstein's relativity, Bulletin of Faculty of Engineering, University of the Ryukyus, Vol.75, pp.1-6, in Japanese (2014).
[7] E. Nakaza, Relativity: What has the physics world proven? Bulletin of Faculty of Engineering, University of the Ryukyus, Vol.76, pp.1-5, in Japanese (2015).