

NNRT's Relativistic Observational Framework Based on Universal Spacetime and the Galilean Transformation

— From the Empirical and Mathematical Derivation of the Lorentz Transformation by Two-Way Light Measurement to Gravitational Observational Scales, GPS, Matter Waves, and Mercury's Perihelion Advance —

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Abstract

In Nakaza's New Relativity Theory (NNRT), universal time and universal space constitute the real foundation, while the real motions of all inertial frames are related by Galilean transformations (GT). We first construct a thought experiment involving the emission, reflection, and reception of light by a uniformly moving rod, starting from records of events made with a common universal clock. The asymmetry of the outward and return propagation times, the classical Doppler relations, and bidirectional measurement records lead, through optical symmetrization, to the Lorentz transformation (LT) of observational time and distance. This empirical construction is given a rigorous mathematical formulation using light-emission and reception events and normalization of the coefficients of the vacuum wave operator. Phase invariance, the relativistic Doppler relation, and the invariance of observational light speed are then established within the same framework. Next, an observational coframe is used to construct the electromagnetic, matter-wave, and clock scales affected by gravitational sources and acceleration, leading to the observational-metric field equations, the Schwarzschild solution, phase transport, gravitational redshift, light deflection, Shapiro delay, and GPS corrections. For matter waves, we extract the action phase from the Schrödinger equation in a Newtonian gravitational field and use the phase equation coupled to the gravitational observational metric to derive an effective potential containing the r^{-3} correction, from which Mercury's perihelion advance is calculated. Finally, we distinguish the parameter of a phase-characteristic curve from universal time and clarify its relationship to a corrected equation of motion constructed on universal spacetime. In this way, NNRT systematically reproduces relativistic observational formulas and numerical results through a reversed direction of theoretical construction.

Keywords: *Nakaza's New Relativity Theory (NNRT); universal time; universal space; Galilean transformation; Lorentz transformation; two-way light measurement; phase invariance; observational metric; gravitational redshift; GPS; matter waves; perihelion advance*

1. Research Objective and Structure of the Framework

In this paper, the direction of physical explanation is defined as proceeding from real motion, wave propagation, phase, and clock-measurement scales on universal spacetime to the observed four-dimensional spacetime. GT specifies coordinate relations for real motion, whereas LT is a mapping of electromagnetic observation constructed upon those relations. In a gravitational field, analogous functions are performed by the observational coframe and observational metric, which couple the null phase characteristics of electromagnetic waves, the action phase of matter waves, and the cycle records of atomic clocks to one field.

The mathematical construction follows the sequence: (i) specification of universal spacetime; (ii) the two-way light-measurement thought experiment; (iii) rigorous derivation of LT as optical observational coordinates; (iv) the gravitational observational-scale field; (v) electromagnetic waves, clocks, and GPS; (vi) matter waves and Mercury's perihelion advance; and (vii) unification of the resulting measurement formulas. In particular, directly recorded times must not be confused with times assigned by an observational mapping, nor should the parameter of real motion be confused with the parameter of a phase-characteristic curve.

2. Foundations of NNRT: Universal Time, Universal Space, and GT

2.1 Definition of Universal Spacetime

The spacetime serving as the foundation of reality is defined by

$$M_{\text{real}} = \mathbb{R}_T \times \mathbb{R}_{\mathbf{X}}^3.$$

For convenience, the correspondence between the variable names and symbols used here and their meanings is summarized in “Appendix A. Definitions of Symbols and Variables” at the end of the paper.

T denotes universal time shared by all inertial frames, and $\mathbf{X}$ denotes position coordinates in universal space. For any two inertial frames S and S' in uniform relative motion with velocity $\mathbf{v}$, real motion is related by GT:

$$T' = T, \quad \boldsymbol{\xi} = \mathbf{X} - \mathbf{v}T.$$

Here, $\boldsymbol{\xi}$ is the moving coordinate. The relation $T' = T$ in this equation is not an approximation; it is the very definition of universal time in NNRT. Calling S the reference frame does not introduce absolute rest or an ether. The frames are distinguished only to describe the arrangement and relative motion of the measurement apparatus.

2.2 Distinction Between Laws of Motion and Rules of Observation

As a basic description of real motion, we begin with the Newtonian form under a force $\mathbf{F}$:

$$m \frac{d^2 \mathbf{X}}{dT^2} = \mathbf{F}$$

For precise orbital observations in gravitational fields, however, an effective correction to the law of motion is required, reflecting the phase transport of matter waves developed below. Observational coordinates $t'_{\text{obs}}, x'_{\text{obs}}$ are constructed separately from $T, \boldsymbol{\xi}$ through optical measurement procedures.

2.3 Distinguishing Axioms, Observational Construction Conditions, and Derived Results

We specify explicitly which elements of the present derivation are axioms, which are construction conditions for electromagnetic measurement, and which are mathematical consequences. Universal time, universal space, and GT are axioms governing real motion; no delay of real time between inertial frames is introduced. Newtonian laws of motion serve as the fundamental equations of real motion described in universal time.

The two-way-light thought experiment uses the propagation law for vacuum electromagnetic waves in a chosen reference inertial frame, records of emission, reflection, and reception events by equivalent universal clocks, and the assignment of observational times to the same events. Constructing a symmetric optical scale by taking the geometric mean of the asymmetric outward and return times actually recorded is an explicit measurement rule applied to measured values; it is not an automatic consequence of GT alone.

The rigorous derivation of the mathematical observational mapping additionally imposes the following conditions: (i) linearity of the mapping for uniform inertial motion; (ii) correspondence between the GT worldline of the moving-frame origin and the optical observational origin; (iii) normalization that makes the coefficients of the same vacuum electromagnetic wave operator identical in the two sets of observational coordinates; and (iv) preservation of orientation in the zero-velocity limit.

$$\mathcal{A}_{\text{real}} + \mathcal{C}_{\text{EM}} \Rightarrow \mathcal{L}_{\text{obs}}$$

Here, $\mathcal{A}_{\text{real}}$ denotes the axioms for real motion based on universal spacetime and GT, $\mathcal{C}_{\text{EM}}$ denotes the electromagnetic observational construction conditions stated above, and $\mathcal{L}_{\text{obs}}$ denotes the resulting LT-type observational mapping. This representation does not presuppose conventional transformation formulas; rather, it enumerates the assumptions needed for a derivation based on the foundational premises and observational procedure of NNRT itself.

Requiring merely that two wave operators have the same set of zero solutions leaves an undetermined nonzero multiplier of the entire operator. The coefficient-normalization condition adopted here is the stronger condition that fixes this freedom. From the LT obtained under this condition and the correspondence of phase to the same event, the simultaneous transformations of frequency and wavenumber, and the invariance of the observed propagation speed of light in vacuum, follow as mathematical theorems.

3. Empirical Thought Experiment: A Moving Rod and Two-Way Light Propagation

3.1 Apparatus, Clocks, and Events

Let the direction of motion be the X axis. A rod of GT-defined length L has rear endpoint A and front endpoint B , and moves with constant velocity v relative to the reference frame. The real worldlines of its endpoints are

$$X_A(T) = vT, \quad X_B(T) = vT + L.$$

At $T = 0$, the transmitter in S coincides with the rear endpoint A . Light is then emitted forward from S , reflected at B , and returns to the still-moving A . The clocks on the rod and in the reference frame all register the same universal time T . Let the universal times of emission, reflection, and reception be $0, T_+, T_+ + T_-$, respectively. Since the return signal is received at the moving endpoint A , this reception differs from one at the transmitter fixed at its initial position.

3.2 Arrival Time Along the Outward Path

The outward light ray follows the vacuum propagation relation $X_{\text{light}} = cT$ in the reference frame. Its intersection condition with the front endpoint is

$$cT_+ = vT_+ + L.$$

Consequently, using $\beta = v/c$, we obtain

$$T_+ = \frac{L}{c - v} = \frac{L/c}{1 - \beta}.$$

This is the universal time required for light to catch up with the receding front endpoint. If the one-way reference time for a stationary rod is $T_0 = L/c$, then $T_+/T_0 = (1 - \beta)^{-1}$.

3.3 Arrival Time Along the Return Path

After reflection, the light propagates leftward while the rear endpoint A moves rightward; their closing speed is $c + v$. Since the rod still has length L in GT coordinates at the moment of reflection,

$$T_- = \frac{L}{c + v} = \frac{L/c}{1 + \beta}.$$

It follows that $T_+ \neq T_-$. The asymmetry arises from the relative motion of the propagating light and the endpoints, not from any change in the rate of universal clocks.

3.4 Round-Trip Time and Two Types of Mean

The round-trip time is

$$\begin{aligned} T_{\text{round}} &= T_+ + T_- \\ &= \frac{L}{c - v} + \frac{L}{c + v} \\ &= \frac{2Lc}{c^2 - v^2} = 2\gamma^2 \frac{L}{c}, \\ \gamma &= \frac{1}{\sqrt{1 - \beta^2}}. \end{aligned}$$

Defining the arithmetic mean of the one-way times as T_b , we obtain

$$T_b = \frac{T_+ + T_-}{2} = \gamma^2 \frac{L}{c}.$$

By contrast, the geometric mean of the same measured values is

$$T_{\text{sym}} = \sqrt{T_+ T_-} = \gamma \frac{L}{c}.$$

Their ratio is $T_{\text{sym}}/T_b = 1/\gamma$. Here, T_{sym} is not either one of the measured one-way times; it is a symmetrized optical scale constructed from the asymmetric bidirectional records. Maintaining this distinction is essential to the rigor of the empirical construction.

4. Wavefront Arrival Intervals and Doppler Relations

4.1 Reception Frequencies Counted by Universal Clocks

Instead of a single pulse, consider a transmitter in the reference frame emitting successive wavefronts at intervals $\Delta T_e = 1/\nu_0$. When a receiver moving rightward with velocity v receives the forward-propagating wavefronts while receding from the transmitter, the reception interval is

$$\Delta T_{\text{rec},+} = \frac{\Delta T_e}{1 - \beta}.$$

Accordingly, the number of wavefronts received per unit universal time is

$$\nu_{T,+} = \nu_0(1 - \beta).$$

For oppositely propagating wavefronts emitted by another transmitter fixed in the reference frame, the corresponding relation is

$$\nu_{T,-} = \nu_0(1 + \beta).$$

These are the classical Doppler relations counted with a common universal clock by a receiver moving relative to transmitters fixed in the reference frame. A wave actually reflected by a moving mirror acquires an additional Doppler factor upon reflection. The transmitter for the opposite direction introduced here is a separate arrangement for comparing bidirectional wavefront arrival rates.

4.2 Correspondence With Light-Propagation Times

For the reception frequencies in the two directions,

$$\frac{\nu_{T,+}}{\nu_{T,-}} = \frac{1 - \beta}{1 + \beta}, \quad \nu_{T,+}\nu_{T,-} = \nu_0^2(1 - \beta^2).$$

Moreover, the propagation times and frequencies obey

$$\frac{T_+}{T_-} = \frac{\nu_{T,-}}{\nu_{T,+}}$$

Thus, the asymmetric propagation times measured in the actual light experiment and the frequency ratio appearing in Doppler observations are related by the same velocity ratio.

4.3 Optical Symmetrization and Relativistic Frequency Ratios

By a symmetrization rule treating the two directions equally and preserving their product, define the times by

$$t'_{\text{obs},+} = T_+ \sqrt{\frac{\nu_{T,+}}{\nu_{T,-}}}, \quad t'_{\text{obs},-} = T_- \sqrt{\frac{\nu_{T,-}}{\nu_{T,+}}}$$

Then both are equal to $\gamma L/c$. This rule does not assert that the real outward and return clock records become equal; it assigns optical times by processing measured data.

Similarly, consider the geometric mean of the bidirectional wavefront arrival rates,

$$\nu_* = \sqrt{\nu_{T,+}\nu_{T,-}} = \frac{\nu_0}{\gamma}$$

Taking this mean as the optical comparison reference gives

$$\frac{\nu_{T,+}}{\nu_*} = \sqrt{\frac{1 - \beta}{1 + \beta}}, \quad \frac{\nu_{T,-}}{\nu_*} = \sqrt{\frac{1 + \beta}{1 - \beta}}.$$

These are the relativistic Doppler factors associated with redshift in the receding direction and blueshift in the approaching direction. By explicitly specifying the construction of the comparison reference ν_* , we avoid conflating classical wavefront arrival rates counted by universal clocks with relativistic optical observational frequencies.

5. Constructing LT From Optical Measurement of Arbitrary Events

5.1 Time Assignment by Light Emission and Reception

Let an observer at the moving rod's origin A send light to an arbitrary event $P = (T, X)$ ahead and receive the light reflected there at A . Introduce the GT coordinate $\xi = X - vT > 0$. If the universal times of emission and reception are T_e, T_r , then the intersections of the light rays with A yield

$$T_e = T - \frac{\xi}{c - v}, \quad T_r = T + \frac{\xi}{c + v}.$$

From these two times, the observer defines the midpoint time and two-way optical distance by

$$T_m = \frac{T_e + T_r}{2}, \quad X_m = \frac{c}{2}(T_r - T_e)$$

Substitution and simplification give

$$\begin{aligned} T_m &= T + \frac{\xi}{2} \left(\frac{1}{c + v} - \frac{1}{c - v} \right) \\ &= T - \gamma^2 \frac{v\xi}{c^2}, \\ X_m &= \frac{c\xi}{2} \left(\frac{1}{c + v} + \frac{1}{c - v} \right) \\ &= \gamma^2 \xi. \end{aligned}$$

For reflection at the rod's front endpoint, $\xi = L$, giving $T_m = \gamma^2 L/c = T_b$. This agrees with the arithmetic mean in the empirical thought experiment.

5.2 Definition of the Observational Scale

The bidirectionally symmetric scale derived empirically is $T_{\text{sym}} = T_b/\gamma$. Apply this normalization uniformly to arbitrary emission-and-reception events, defining

$$t'_{\text{obs}} = \frac{T_m}{\gamma}, \quad x'_{\text{obs}} = \frac{X_m}{\gamma}$$

Using the same factor for both coordinates is a construction condition for the observational mapping and corresponds to normalization of the wave-operator coefficients developed below. Consequently,

$$t'_{\text{obs}} = \frac{T}{\gamma} - \gamma \frac{v\xi}{c^2}, \quad x'_{\text{obs}} = \gamma\xi.$$

Substituting $\xi = X - vT$ yields

$$t'_{\text{obs}} = \gamma \left(T - \frac{vX}{c^2} \right), \quad x'_{\text{obs}} = \gamma(X - vT).$$

LT thus appears as a mapping of electromagnetic observation, distinct from the real-motion transformation on universal spacetime.

6. Rigorous Derivation of LT by Normalization of the Wave Operator

6.1 Linear Observational Mapping and the Origin Condition

Spatial and temporal homogeneity motivate a linear observational mapping. We impose the condition that the GT worldline $X = vT$ of the moving origin maps to $x'_{\text{obs}} = 0$. The most general one-dimensional form is

$$x'_{\text{obs}} = a(X - vT), \quad t'_{\text{obs}} = bT + dX$$

where a, b, d are functions of v .

6.2 Wave Equation With Identically Normalized Coefficients

Take the principal vacuum electromagnetic propagation equation in the reference frame as

$$\left(\frac{1}{c^2} \frac{\partial^2}{\partial T^2} - \frac{\partial^2}{\partial X^2} \right) \psi = 0$$

We require the observational coordinates to preserve the same wave operator, including its overall multiplicative coefficient.

The differential operators are

$$\partial_T = b \partial_{t'} - av \partial_{x'}, \quad \partial_X = d \partial_{t'} + a \partial_{x'}$$

Substitution and expansion yield

$$\begin{aligned} \square &= \left(\frac{b^2}{c^2} - d^2 \right) \partial_{t'}^2 \\ &\quad - 2a \left(\frac{bv}{c^2} + d \right) \partial_{t'} \partial_{x'} \\ &\quad - a^2 (1 - \beta^2) \partial_{x'}^2. \end{aligned}$$

The condition that the mixed-derivative term vanish is

$$d = -\frac{bv}{c^2}.$$

Requiring the coefficients of the temporal and spatial terms to be c^{-2} and -1 , respectively, yields

$$b^2(1 - \beta^2) = 1, \quad a^2(1 - \beta^2) = 1.$$

Imposing continuous recovery of orientation as $v \rightarrow 0$ and selecting the positive branch gives

$$a = b = \gamma, \quad d = -\gamma \frac{v}{c^2}.$$

Therefore, exactly the same LT as the observational mapping of the previous section is uniquely obtained. Crucially, invariance of light speed is not independently assumed as a second postulate here; rather, the physical condition of coefficient normalization of the wave operator is explicitly stated. Preservation merely of the solution set of a wave equation leaves an arbitrary overall factor and does not determine the coefficients uniquely without this normalization condition.

6.3 Phase Invariance and the Theorem of Observational Light Speed

Write the phase of the same wavefront as

$$\theta = \omega T - kX = \omega' t'_{\text{obs}} - k' x'_{\text{obs}}$$

Inserting LT and equating coefficients gives

$$\omega' = \gamma(\omega - vk), \quad k' = \gamma \left(k - \frac{v\omega}{c^2} \right).$$

It follows that

$$\omega'^2 - c^2 k'^2 = \omega^2 - c^2 k^2.$$

From the vacuum dispersion relation $\omega^2 - c^2 k^2 = 0$ of the reference wave, the observational frame likewise has $|\omega'/k'| = c$. The invariance of light speed in a moving observational frame therefore follows from the conditions of the electromagnetic observational mapping.

For forward propagation, taking $k = \omega/c$ gives

$$\frac{v'}{v} = \gamma(1 - \beta) = \sqrt{\frac{1 - \beta}{1 + \beta}}.$$

The factor for the opposite direction is $v'/v = \gamma(1 + \beta)$. These results agree with the factors constructed from the empirical bidirectional Doppler measurements.

6.4 Phase Change per Universal Time and Observed Frequency

Along the moving observer's GT worldline $X = vT$, the rate of phase change is

$$\frac{d\theta}{dT} = \omega - vk.$$

On the same worldline, the LT observational time satisfies $dt'_{\text{obs}}/dT = 1/\gamma$, so that

$$\frac{d\theta}{dt'_{\text{obs}}} = \gamma(\omega - vk) = \omega'.$$

This distinction is essential. Although the phase θ is invariant for a given wavefront event, its temporal and spatial gradients depend on measurement scales. Received phase and observed frequency are distinguished without changing the flow of universal time.

7. Generalization to Gravity: Observational Coframe and Field Equations

7.1 Observational Scale Field Constructed on Universal Spacetime

Write the real coordinates as $X^\mu = (cT, X^1, X^2, X^3)$ and introduce an observational coframe $e_\mu^a(X)$. Define the local observational-scale one-forms as $\vartheta^a = e_\mu^a dX^\mu$ and construct

$$g_{\mu\nu}^{\text{obs}} = \eta_{ab} e_\mu^a e_\nu^b, \quad \eta_{ab} = \text{diag}(-1, 1, 1, 1)$$

The observational metric represents the measurement relations of clocks, light, and matter waves, and is distinct from the definition of universal spacetime itself. A spatially varying observational coframe is not generally integrable into ordinary coordinate differentials; it can therefore yield a nontrivial observational geometry.

7.2 Gravitational Action for the Observational Metric

Denote the torsion of the observational coframe by $T_{\mu\nu}^\rho$ and its trace by T_ρ . Define the torsion scalar as

$$\mathcal{T} = \frac{1}{4} T^{\rho\mu\nu} T_{\rho\mu\nu} + \frac{1}{2} T^{\rho\mu\nu} T_{\nu\mu\rho} - T_{\rho} T^{\rho}$$

For a coframe with an appropriate flat inertial connection, the geometric identity reads

$$R[g_{\text{obs}}] = -\mathcal{T} + B$$

Here, B is a total-divergence term. In geometrized units $c = 1$, construct the action

$$S_{\text{obs}} = -\frac{1}{2\kappa} \int e \mathcal{T} d^4X + S_{\text{matter}}, \quad \kappa = 8\pi G, \quad e = \det(e_{\mu}^a)$$

With suitable boundary conditions specified in its variation, the resulting equation is

$$G_{\mu\nu}[g_{\text{obs}}] = \frac{8\pi G}{c^4} T_{\mu\nu}^{\text{obs}}$$

In this paper, we take this equation as the field equation for the observational-scale field of NNRT. It enables known exact solutions for observational geometry to be derived with the same mathematical relations. The gravitational action and common coupling to matter are stated explicitly as construction conditions of NNRT's gravitational sector.

7.3 Common Coupling of Electromagnetic Waves, Matter Waves, and Clocks to the Observational-Scale Field

To derive not only the field equations but also the measurement records of electromagnetic waves, matter waves, and clocks from a common observational-scale field, we make the coupling laws for their respective degrees of freedom explicit. The following decomposition of the action unifies the field equations and phase-transport equations introduced thus far within a single construction.

$$S_{\text{total}} = S_{\text{obs}}[e] + S_{\text{EM}}[A, e] + S_{\text{m}}[\psi, e] + S_{\text{clock}}[e, \theta_{\text{clock}}]$$

Here, e denotes the observational coframe, A the electromagnetic potential, ψ the matter-wave field, and θ_{clock} the internal oscillation phase of a clock. Through the same observational coframe, these fields share local observational scales of time and space.

For the electromagnetic field we introduce a kinetic term defined by the observational metric, whose geometric-optics limit yields the null-characteristic equation for phase.

$$S_{\text{EM}} = -\frac{1}{4} \int e F_{\mu\nu} F^{\mu\nu} d^4X$$

$$g_{\text{obs}}^{\mu\nu} (\partial_{\mu} \theta) (\partial_{\nu} \theta) = 0$$

The matter-wave field is coupled to the wave operator of the same metric. In its short-wavelength limit, this yields the Hamilton–Jacobi equation containing a mass term. At this stage, electromagnetic and matter waves have different characteristic equations while employing the same observational-scale field.

$$\left(\square_{\text{obs}} - \frac{m^2 c^2}{\hbar^2} \right) \psi = 0$$

$$g_{\text{obs}}^{\mu\nu} (\partial_{\mu} S) (\partial_{\nu} S) + m^2 c^2 = 0$$

For clocks, the period count N of an oscillator with a local reference frequency ν_0 is related to the observational scale factor C . This factor is determined by the gravitational source, state of motion, and observational location; specific forms for stationary clocks and clocks on circular orbits are derived in subsequent sections.

$$\theta_{\text{clock}}(T) = 2\pi\nu_0 \int^T C(\mathbf{X}, \mathbf{V}, \Phi) dT$$

$$\frac{dN}{dT} = \nu_0 C(\mathbf{X}, \mathbf{V}, \Phi)$$

Coupling these equations to the same observational coframe is adopted as a construction condition of the gravitational-observation sector. In other words, the field equations for the observational metric alone are not assumed to produce the microscopic oscillation law of an atomic clock automatically; rather, the coupling of phase records is itself adopted as an explicit law of the theory. This enables gravitational redshift, GPS frequency calibration, and orbital corrections arising from matter waves to be described within a common mathematical normalization.

8. Spherically Symmetric Observational Scales and the Schwarzschild Solution

8.1 General Static, Spherically Symmetric Form

Take a static, spherically symmetric observational metric of the form

$$ds_{\text{obs}}^2 = -A(r)c^2dT^2 + B(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

The independent components of the vacuum field equation $G_{\mu\nu}[g_{\text{obs}}] = 0$ give

$$\frac{d}{dr} \left[r \left(1 - \frac{1}{B(r)} \right) \right] = 0, \quad \frac{d}{dr} \ln[A(r)B(r)] = 0$$

Imposing the asymptotic normalization $A(\infty) = B(\infty) = 1$ and the weak-field normalization for mass M yields

$$A(r) = 1 - \frac{2GM}{rc^2}, \quad B(r) = A(r)^{-1}.$$

Defining $r_s = 2GM/c^2$ and $f(r) = 1 - r_s/r$, we obtain

$$ds_{\text{obs}}^2 = -f(r)c^2dT^2 + f(r)^{-1}dr^2 + r^2d\Omega^2.$$

We use this as the Schwarzschild observational metric. Throughout the following analysis, the exterior region $r > r_s$ is considered. The choice T here identifies the static coordinate time normalized at infinity with the reference for universal time; the rule governing this coordinate correspondence is thus made explicit.

8.2 Stationary Clocks and Observational Scales

For a clock stationary at position r , we have $dr = d\theta = d\phi = 0$, and hence

$$d\tau_{\text{obs}} = C_g(r)dT, \quad C_g(r) = \sqrt{f(r)} = \sqrt{1 - \frac{2GM}{rc^2}}.$$

Moreover,

$$c^2 C_g \frac{dC_g}{dr} = \frac{GM}{r^2}$$

This links the magnitude of the acceleration due to the gravitational source to the spatial gradient of the observational time scale. The quantity τ_{obs} is the clock reading based on a measurement scale, not universal time T itself.

9. Electromagnetic Phase, Geodesics, and Gravitational Redshift

9.1 Phase Equation and Transport Equations

Suppressing the constant prefactor dependent on electromagnetic units, write the action of the electromagnetic field coupled to the observational metric as

$$S_{\text{EM}} = -\frac{1}{4} \int e F_{\mu\nu} F^{\mu\nu} d^4X$$

In the geometrical-optics limit of the corresponding wave equation, the electromagnetic phase gradient $k_\mu = \partial_\mu \theta$ satisfies

$$g_{\text{obs}}^{\mu\nu} k_\mu k_\nu = 0$$

Writing $H = 1/2 g_{\text{obs}}^{\mu\nu} k_\mu k_\nu$ yields

$$\frac{dX^\mu}{d\lambda} = g_{\text{obs}}^{\mu\nu} k_\nu, \quad \frac{dk_\mu}{d\lambda} = -\frac{1}{2} \partial_\mu g_{\text{obs}}^{\alpha\beta} k_\alpha k_\beta$$

Here, λ is an affine parameter along a phase-characteristic curve. Light rays follow null geodesics of the observational metric. NNRT assigns this result the role of a propagation law for electromagnetically observed phase in the presence of a gravitational source.

9.2 Radial Phase and Shapiro-Type Propagation

The radial phase equation is

$$-\frac{1}{f(r)c^2} \left(\frac{\partial \theta}{\partial T} \right)^2 + f(r) \left(\frac{\partial \theta}{\partial r} \right)^2 = 0.$$

Separating variables by static symmetry with $\theta = -\omega_\infty T + W(r)$ gives

$$\frac{dW}{dr} = \pm \frac{\omega_\infty}{cf(r)}.$$

Integration yields

$$\theta = -\omega_\infty T \pm \frac{\omega_\infty}{c} \left[r + r_s \ln \left(\frac{r}{r_s} - 1 \right) \right] + \text{const.}$$

The constant-phase condition $d\theta = 0$ gives the coordinate propagation speed $dr/dT = \pm cf(r)$. By contrast, the local observational scales $d\ell_{\text{obs}} = dr/\sqrt{f}$ and $d\tau_{\text{obs}} = \sqrt{f} dT$ give the measured light speed $d\ell_{\text{obs}}/d\tau_{\text{obs}} = c$. Coordinate propagation in a gravitational field must be distinguished from light speed measured with local scales.

9.3 Gravitational Redshift and Blueshift

In the static observational metric, $\omega_\infty = -\partial_T \theta$ is conserved along a ray. By contrast, a stationary observer at radius r counts frequency in local observational time as

$$\omega_{\text{obs}}(r) = -\frac{d\theta}{d\tau_{\text{obs}}} = \frac{\omega_\infty}{\sqrt{f(r)}}$$

Thus, for emission at r_e and reception at r_r ,

$$\frac{v_r}{v_e} = \sqrt{\frac{1 - 2GM/(r_e c^2)}{1 - 2GM/(r_r c^2)}}.$$

For $r_r > r_e$, redshift occurs; propagation in the opposite direction yields blueshift. In the weak-field approximation using the Newtonian potential $\Phi(r) = -GM/r$,

$$\frac{v_r - v_e}{v_e} \simeq \frac{\Phi(r_e) - \Phi(r_r)}{c^2}.$$

Here, too, any statement that “the phase itself changes” must distinguish phase invariance for the same wavefront from changes of phase gradient relative to the measured time scale.

10. Clock Oscillation Phase and Measurement Records

Let ν_0 be the reference frequency of an atomic clock normalized locally. In the measurement sector of NNRT, construct its observational phase as

$$\theta_{\text{clock}}(T) = 2\pi\nu_0 \int^T C(\mathbf{X}, \mathbf{V}, \Phi) dT$$

If $N = \theta_{\text{clock}}/(2\pi)$ denotes the accumulated cycle count, then

$$\frac{dN}{dT} = \nu_0 C(\mathbf{X}, \mathbf{V}, \Phi).$$

For a clock stationary in a spherically symmetric gravitational field, $C = C_g = \sqrt{f}$; the ratio of cycle-recording rates of two stationary clocks is therefore

$$\frac{dN_e/dT}{dN_r/dT} = \sqrt{\frac{f(r_e)}{f(r_r)}}.$$

In addition to the wave propagation equation, this clock-phase coupling law relates the actual readings of measuring instruments to a common observational scale. A constant propagation delay in a static gravitational field does not, by itself, necessarily produce a frequency difference when compared with uniform universal clocks. This is why the local phase scale of the clock must be specified separately.

11. Observational Formulas for Light Deflection and Shapiro Delay

11.1 Deflection of Light

For the null phase characteristics of the Schwarzschild observational metric, put $u = 1/r$ in the equatorial plane. Expressing the orbital equation in terms of the conserved angular momentum and frequency of light yields

$$\frac{d^2 u}{d\phi^2} + u = \frac{3GM}{c^2} u^2.$$

Inserting the zeroth-order solution $u_0 = \sin\phi/b$, where b is the impact parameter, into the right-hand side, a first-order perturbative solution gives the difference between incoming and outgoing directions at infinity:

$$\Delta\theta = \frac{4GM}{bc^2} + O\left(\frac{G^2M^2}{b^2c^4}\right).$$

Using the leading terms for a grazing ray at the solar limb $b \simeq R_\odot$ gives approximately $1.75''$. This result follows directly from the null characteristic curves of the observed phase.

11.2 Shapiro Delay

Integration of the phase characteristics of the observational metric gives the first-order gravitational propagation delay. Let the radial coordinates of the emitting and receiving locations be $\mathbf{r}_e, \mathbf{r}_r$ and their Euclidean separation be R . Then

$$\Delta T_{\text{Shapiro}} \simeq \frac{2GM}{c^3} \ln \left(\frac{r_e + r_r + R}{r_e + r_r - R} \right).$$

This expression represents the influence of the gravitational source on the phase gradient and propagation path of light. In GPS and other measurements, it is distinguished from the Sagnac contribution and propagation corrections arising from the atmosphere and ionosphere.

12. GPS: Constructing Observational Clock Scales From Gravity and Centripetal Acceleration

12.1 Circular Motion Is Not Uniform-Velocity Inertial Motion

In universal space, express a circular orbit of radius r and angular velocity Ω by

$$\mathbf{X}(T) = r(\cos\Omega T, \sin\Omega T, 0)$$

Although the speed $V = \Omega r$ is constant, the velocity vector changes, giving

$$\frac{d^2\mathbf{X}}{dT^2} = -\Omega^2\mathbf{X}, \quad a_c = \Omega^2 r = \frac{V^2}{r}$$

Here, a_c is the coordinate centripetal acceleration relative to Earth's center, not the proper acceleration measured by an accelerometer onboard an ideal freely falling satellite. NNRT does not introduce the GPS time correction as a delay of real time between uniformly moving inertial frames. Instead, the correction is calculated using observational scales incorporating the gravitational source and circular motion.

12.2 Rotating Observational Coordinates and Time Scales

In the equatorial plane, the Schwarzschild observational metric is

$$ds_{\text{obs}}^2 = -f(r)c^2dT^2 + f(r)^{-1}dr^2 + r^2d\phi^2.$$

Introducing a rotating observational coordinate $\chi = \phi - \Omega T$ gives $d\phi = d\chi + \Omega dT$ and hence

$$ds_{\text{obs}}^2 = -[f(r)c^2 - \Omega^2r^2]dT^2 + 2\Omega r^2d\chi dT + r^2d\chi^2 + f(r)^{-1}dr^2.$$

At an observation point corotating with the satellite, $dr = d\chi = 0$, yielding

$$C_{\text{orb}}(r) = \frac{d\tau_{\text{obs}}}{dT} = \sqrt{1 - \frac{2GM}{rc^2} - \frac{\Omega^2r^2}{c^2}}.$$

Using $a_c = \Omega^2 r$, we obtain

$$C_{\text{orb}}(r) = \sqrt{1 - \frac{2GM}{rc^2} - \frac{a_c r}{c^2}}.$$

The two terms in this expression represent the scale due to the gravitational source and the scale associated with the acceleration of circular motion. The latter need not be interpreted as a delay of universal time caused by LT.

12.3 Observational Potential and the Circular-Orbit Condition

Using the gravitational potential $\Phi_g = -GM/r$ and the inertial potential $\Phi_c = -\Omega^2r^2/2$ of rotating coordinates gives

$$C_{\text{orb}} = \sqrt{1 + \frac{2(\Phi_g + \Phi_c)}{c^2}}.$$

The quantity Φ_c is an inertial potential of rotating coordinates, not an independent gravitational source. Define the circular-orbit matter-wave action $S_m = -mc^2 \int C_{\text{orb}} dT$, and for a radial variation at fixed orbital radius impose

$$\frac{\partial}{\partial r} \left[f(r) - \frac{\Omega^2 r^2}{c^2} \right] = 0$$

The resulting condition is

$$\frac{2GM}{r^2 c^2} - \frac{2\Omega^2 r}{c^2} = 0, \quad \Omega^2 r = \frac{GM}{r^2}.$$

Thus $a_c = GM/r^2$. Substituting this into the clock scale gives

$$C_{\text{GPS}}(r) = \sqrt{1 - \frac{3GM}{rc^2}}$$

The quantity $3GM/(rc^2)$ under the square root is composed of the gravitational observational-scale term $2GM/(rc^2)$ and the circular-acceleration term $GM/(rc^2)$.

12.4 Frequency Ratio Relative to a Ground Clock

Let the terrestrial rotation rate be Ω_E , the geocentric radius of a ground clock be r_g , and its latitude be λ . Under the spherical approximation, the observational scale of the ground clock is

$$C_{\text{ground}} = \sqrt{1 - \frac{2GM}{r_g c^2} - \frac{\Omega_E^2 r_g^2 \cos^2 \lambda}{c^2}}.$$

When the same reference frequency ν_0 is used for the satellite and ground clocks, their phase-recording-rate ratio per unit universal time is

$$\frac{dN_{\text{sat}}}{dN_{\text{ground}}} = \frac{\sqrt{1 - 3GM/(r_{\text{sat}} c^2)}}{\sqrt{1 - 2GM/(r_g c^2) - \Omega_E^2 r_g^2 \cos^2 \lambda / c^2}}.$$

Expanding in the weak field through order c^{-2} gives

$$\frac{\Delta\nu}{\nu} \simeq \frac{GM}{c^2} \left(\frac{1}{r_g} - \frac{1}{r_{\text{sat}}} \right) - \frac{a_c r_{\text{sat}} - \Omega_E^2 r_g^2 \cos^2 \lambda}{2c^2}.$$

In this representation, the first term arises from the gravitational source and the second from observational scales associated with circular acceleration of the satellite and the ground station. Gravity and centripetal acceleration are not two independent forces in a freely falling circular orbit; they are connected by the circular-orbit condition and must not be counted twice.

12.5 Numerical Verification of GPS Corrections

Use $GM = 3.986004418 \times 10^{14} \text{ m}^3 \text{ s}^{-2}$, $r_{\text{sat}} = 2.6560 \times 10^7 \text{ m}$, $r_g = 6.378137 \times 10^6 \text{ m}$, and $\Omega_E = 7.2921150 \times 10^{-5} \text{ rad s}^{-1}$, together with an equatorial ground clock ($\lambda = 0$). The satellite's centripetal acceleration is

$$a_c = \frac{GM}{r_{\text{sat}}^2} \simeq 0.56504 \text{ m s}^{-2}.$$

Taking one day as 86400 s and converting these relations into daily clock differences, the gravitational scale difference is approximately $+45.651 \mu\text{s/day}$, and the circular-motion scale difference is approximately $-7.110 \mu\text{s/day}$. Their total is

$$\Delta t_{\text{net}} \simeq +38.541 \mu\text{s/day}.$$

This result uses a spherically symmetric Earth, a circular orbit, and an equatorial reference clock. Operational GPS uses a convention including Earth's oblateness and geoid; its mean fractional frequency correction is approximately 4.4647×10^{-10} , corresponding to approximately $38.6 \mu\text{s}$ per day. Presetting the satellite's transmitted frequency to a prescribed value amounts to accounting for the difference between observational clock scales in the instrument's oscillation record.

12.6 Clock Phase on Elliptical Orbits

For a noncircular orbit, to leading order in the weak-field and low-speed approximation, put $V^2 = (d\mathbf{X}/dT)^2$ and use

$$\frac{dN}{dT} = v_0 \sqrt{1 - \frac{2GM}{rc^2} - \frac{V^2}{c^2}}$$

A first-order weak-field expansion and the elliptical-orbit energy relation $V^2 = GM(2/r - 1/a)$ yield

$$\frac{1}{v_0} \frac{dN}{dT} \simeq 1 - \frac{2GM}{rc^2} + \frac{GM}{2ac^2}.$$

Integrating the difference from the mean circular-orbit scale along Keplerian motion, the variable clock term is

$$\delta t_{\text{clock}} = -\frac{2e\sqrt{GMa}}{c^2} \sin E + \text{const.}$$

Here, a is the semimajor axis, e the orbital eccentricity, and E the eccentric anomaly. Corrections applied at the receiver must respect the sign relative to the variation of the transmitting clock.

13. Signal Phase and Reception Records in GPS

GPS positioning does not consist solely of comparing clocks. It involves receiving the electromagnetic signal transmitted from a satellite and comparing its phase with that of a local clock. The emission event P_e and reception event P_r are related through the phase correspondence of the same wavefront,

$$\theta_{\text{photon}}(P_e) = \theta_{\text{photon}}(P_r)$$

The quantity recorded by the receiver is, for example,

$$\Delta\theta_{\text{meas}} = \theta_{\text{photon},r} - \theta_{\text{clock},r}$$

This is the measured phase difference. The time relation between emission and reception also incorporates geometric propagation time, the Sagnac effect, Shapiro delay, and propagation corrections due to the ionosphere and troposphere. The first-order Sagnac term arising from rotation of Earth-fixed coordinates can be written, depending on the coordinate-sign convention, as

$$\Delta T_{\text{Sagnac}} \simeq \frac{\Omega_E}{c^2} (x_e y_r - y_e x_r)$$

Computing clock scales and optical propagation phase separately and combining them at the receiver clarifies the connection between NNRT and actual measurements.

14. Introducing Matter Waves: From Newtonian Motion to Phase Transport

14.1 Motion in Newtonian Gravity and Its Limits

Let the solar mass be M and Mercury's mass be m . Newtonian motion in universal time is described by

$$m \frac{d^2 \mathbf{X}}{dT^2} = -\frac{GMm}{r^2} \hat{\mathbf{r}}$$

With specific angular momentum $h_T = r^2 d\phi/dT$, the classical energy is

$$E_N = \frac{m}{2} \left(\frac{dr}{dT} \right)^2 + \frac{mh_T^2}{2r^2} - \frac{GMm}{r}.$$

This gravitational field already has a spatial gradient through r , but the inverse-square law alone yields no secular perihelion advance around an isolated spherical source. The required correction is not attributable merely to the existence of a gradient. It is constructed through a more precise transport law for the action phase of matter waves in the gravitational field.

14.2 The Schrödinger Equation and Action Phase

Write nonrelativistic quantum mechanics in universal time as

$$i\hbar \frac{\partial \psi}{\partial T} = \left(-\frac{\hbar^2}{2m} \nabla^2 - \frac{GMm}{r} \right) \psi$$

and put $\psi = A \exp(iS/\hbar)$. Retaining the leading-order terms of $\hbar$ gives

$$\frac{\partial S}{\partial T} + \frac{(\nabla S)^2}{2m} - \frac{GMm}{r} = 0$$

This is the Newtonian Hamilton–Jacobi equation, which by itself reproduces only closed Keplerian orbits. The following correction does not arise automatically from the Schrödinger equation alone; it is obtained by coupling matter waves to the gravitational observational metric constructed in preceding sections.

14.3 Matter-Wave Equation Coupled to the Observational-Scale Field

Take the equation for a scalar matter wave coupled to the observational metric to be

$$\left(\square_{\text{obs}} - \frac{m^2 c^2}{\hbar^2} \right) \psi = 0$$

In the short-wavelength limit, its phase $\psi = A \exp(iS/\hbar)$ satisfies

$$g_{\text{obs}}^{\mu\nu} \partial_\mu S \partial_\nu S + m^2 c^2 = 0$$

This Hamilton–Jacobi equation governs the observed phase of matter waves and uses the same observational metric as the previously derived null-phase equation for electromagnetic waves.

15. Deriving the r^{-3} Correction From the Matter-Wave Phase Equation

15.1 Separated Action and Conserved Quantities

In the equatorial plane $\theta = \pi/2$, separate the matter-wave action phase as

$$S = -\mathcal{E}T + J\phi + W(r)$$

Here, $\mathcal{E}$ is the conserved phase energy associated with time-translation symmetry, and J is the phase angular momentum. In what follows, define

$$\varepsilon = \frac{\mathcal{E}}{mc^2}, \quad h = \frac{J}{m}$$

Then, in the Schwarzschild observational metric,

$$-\frac{\varepsilon^2}{f(r)c^2} + f(r) \left(\frac{dW}{dr} \right)^2 + \frac{J^2}{r^2} + m^2 c^2 = 0.$$

Rearranging and multiplying by $f(r)$ yields

$$f(r)^2 \left(\frac{dW}{dr} \right)^2 = \frac{\varepsilon^2}{c^2} - f(r) \left(m^2 c^2 + \frac{J^2}{r^2} \right).$$

15.2 Parameter of the Phase-Characteristic Curve

Introduce a parameter σ along the phase-characteristic curve of the matter wave, and define

$$\frac{dr}{d\sigma} = \frac{f(r)}{m} \frac{dW}{dr}, \quad \frac{d\phi}{d\sigma} = \frac{h}{r^2}$$

This σ is not universal time T , but a parameter used to describe the phase-characteristic curve. Substituting the preceding relations gives

$$\left(\frac{dr}{d\sigma} \right)^2 = c^2 \varepsilon^2 - f(r) \left(c^2 + \frac{h^2}{r^2} \right).$$

Expanding $f(r) = 1 - 2GM/(rc^2)$ gives

$$\begin{aligned} \left(\frac{dr}{d\sigma} \right)^2 &= c^2 (\varepsilon^2 - 1) + \frac{2GM}{r} \\ &\quad - \frac{h^2}{r^2} + \frac{2GMh^2}{c^2 r^3}. \end{aligned}$$

Multiplying both sides by $m/2$ and rearranging gives

$$\frac{m}{2} \left(\frac{dr}{d\sigma} \right)^2 + \frac{mh^2}{2r^2} - \frac{GMm}{r} - \frac{GMmh^2}{c^2 r^3} = \frac{mc^2}{2} (\varepsilon^2 - 1).$$

Consequently,

$$U_{\text{eff}}(r) = \frac{mh^2}{2r^2} - \frac{GMm}{r} - \frac{GMmh^2}{c^2 r^3}.$$

The first term is the centrifugal barrier, the second is Newtonian gravity, and the third is the r^{-3} correction that arises from matter-wave phase transport coupled to the gravitational observational scale.

16. Corrected Orbital Equation and Mercury's Perihelion Advance

16.1 Orbital Equation in Terms of $u = 1/r$

Putting $u = 1/r$ and using $d\phi/d\sigma = h/r^2 = hu^2$ gives

$$\frac{dr}{d\sigma} = \frac{dr}{d\phi} \frac{d\phi}{d\sigma} = -h \frac{du}{d\phi},$$

$$\frac{d^2 r}{d\sigma^2} = -h^2 u^2 \frac{d^2 u}{d\phi^2}.$$

Differentiating the effective energy equation gives

$$m \frac{d^2 r}{d\sigma^2} = -\frac{dU_{\text{eff}}}{dr}.$$

Explicitly,

$$-\frac{1}{m} \frac{dU_{\text{eff}}}{dr} = \frac{h^2}{r^3} - \frac{GM}{r^2} - \frac{3GMh^2}{c^2 r^4}.$$

Substituting this into the expression involving u and dividing by $-h^2 u^2$ gives

$$\frac{d^2 u}{d\phi^2} + u = \frac{GM}{h^2} + \frac{3GM}{c^2} u^2.$$

The difference from the Newtonian orbital equation is its final term, $3GMu^2/c^2$, on the right-hand side.

16.2 Perturbative Calculation Without Skipping the Intermediate Steps

Defining the semilatus rectum as $p = h^2/(GM)$, the orbital equation becomes

$$u'' + u = \frac{1}{p} + \frac{3GM}{c^2} u^2, \quad ' = \frac{d}{d\phi}.$$

The zeroth-order Newtonian ellipse is $u_0 = (1 + e \cos \phi)/p$. Substituting $u = u_0 + \delta u$ and retaining terms through first order in $GM/(c^2 p)$ gives

$$\delta u'' + \delta u = \frac{3GM}{c^2 p^2} (1 + 2e \cos \phi + e^2 \cos^2 \phi).$$

Find a particular solution for the resonant $\cos \phi$ term. Using the identity

$$\left(\frac{d^2}{d\phi^2} + 1 \right) (\phi \sin \phi) = 2 \cos \phi$$

we obtain the secular term

$$\delta u_{\text{sec}} = \frac{3GMe}{c^2 p^2} \phi \sin \phi.$$

For a small $\delta = 3GM/(c^2 p)$, expand

$$\cos[(1 - \delta)\phi] \simeq \cos \phi + \delta \phi \sin \phi$$

The corrected orbit can therefore be written as

$$u \simeq \frac{1}{p} \{1 + e \cos[(1 - \delta)\phi]\}$$

The next perihelion occurs at the angle $\phi = 2\pi/(1 - \delta)$; hence the advance per revolution is

$$\Delta\varpi = \frac{2\pi}{1 - \delta} - 2\pi$$

$$\Delta\varpi \simeq 2\pi\delta = \frac{6\pi GM}{c^2 p}.$$

Using $p = a(1 - e^2)$ for a Keplerian ellipse gives

$$\Delta\varpi = \frac{6\pi GM}{a(1 - e^2)c^2}.$$

16.3 Numerical Evaluation for Mercury

Using $GM_{\odot} = 1.32712440018 \times 10^{20} \text{ m}^3 \text{ s}^{-2}$, $a = 5.790905 \times 10^{10} \text{ m}$, $e = 0.205630$, and $c = 299792458 \text{ m s}^{-1}$ gives

$$p = a(1 - e^2) \simeq 5.546044 \times 10^{10} \text{ m}.$$

The excess angle per revolution is

$$\Delta\varpi \simeq 5.018663 \times 10^{-7} \text{ rad/orbit}.$$

Taking Mercury's orbital period as approximately 87.9691 day, multiplying by the number of revolutions in 100 Julian years (36525 day), and converting with $1 \text{ rad} = 206264.806 \text{ arcsec}$ gives

$$\Delta\varpi_{\text{Mercury}} \simeq 42.981 \text{ arcsec/century}.$$

This agrees numerically with the leading relativistic perihelion-advance contribution of the Sun's spherically symmetric gravitational field. It is distinguished from the full observed precession, which also contains planetary perturbations and contributions from solar nonsphericity.

16.4 Independent Verification of the Perihelion Advance From the Radial Action of Matter Waves Over One Period

In addition to the perturbative solution of the orbital equation obtained above, we verify that the same perihelion advance follows directly from matter-wave phase by evaluating the radial action over one complete period. We use the canonical radial momentum while retaining the radial component of the observational metric. This calculation is distinct from substituting only the r^{-3} term in the effective potential into an action integral.

We put $\mu = GM$ and separate the matter-wave action of the preceding section as $S = -\mathcal{E}T + J\phi + W(r)$. The phase-characteristic angular momentum is $h = J/m$, and the conserved energy is parametrized by $\varepsilon = \mathcal{E}/(mc^2)$. The canonical radial momentum and the radial action over one complete period are then

$$p_r = \frac{dW}{dr} = \frac{m}{f(r)} \sqrt{\varepsilon^2 c^2 - f(r) \left(c^2 + \frac{h^2}{r^2} \right)}$$

$$J_r(\varepsilon, J) = \oint p_r dr, \quad f(r) = 1 - \frac{2\mu}{rc^2}$$

respectively. Hereafter, the radial action J_r and the conserved phase angular momentum J are distinct physical quantities. We define the specific energy ϵ_N in the weak-field limit and the zeroth-order radial momentum as follows:

$$\epsilon_N = \frac{c^2}{2}(\varepsilon^2 - 1), \quad Q_0 = 2\epsilon_N + \frac{2\mu}{r} - \frac{h^2}{r^2}$$

$$p_{r0} = m\sqrt{Q_0}$$

Using $f(r)^{-1} = 1 + 2\mu/(rc^2) + O(c^{-4})$, and expanding both the square root and its prefactor consistently to first order, we obtain

$$p_r = m\sqrt{Q_0} + \frac{m}{c^2} \left[\frac{2\mu}{r} \sqrt{Q_0} + \frac{\mu h^2}{r^3 \sqrt{Q_0}} \right] + O(c^{-4})$$

In this expansion, the first of the two correction terms arises from the inverse radial metric factor $f(r)^{-1}$. The other term represents the coupling between the gravitational field and angular momentum. Both contributions must be included when calculating the correction to the radial action over one complete period.

$$\delta J_r = \frac{m}{c^2} \oint \left[\frac{2\mu\sqrt{Q_0}}{r} + \frac{\mu h^2}{r^3 \sqrt{Q_0}} \right] dr$$

At zeroth order, $Q_0 = (dr/dT_N)^2$. The time parameter T_N refers to the reference Newtonian orbit used to evaluate this integral and coincides, at zeroth order, with motion in the universal time T of NNRT. Taking the integration contour from perihelion to aphelion and back, and denoting the period by P , we have

$$\delta J_r = \frac{m}{c^2} \left[2\mu \int_0^P \frac{\dot{r}_N^2}{r} dT_N + \mu h^2 \int_0^P \frac{dT_N}{r^3} \right]$$

For the zeroth-order elliptical orbit, with semi-latus rectum $p = h^2/\mu$ and eccentricity e , we may use $r = p/(1 + e\cos\phi)$, $dT_N = r^2 d\phi/h$, and $\dot{r}_N = (\mu e/h)\sin\phi$.

$$\int_0^P \frac{\dot{r}_N^2}{r} dT_N = \frac{2\pi\mu}{h} (1 - \sqrt{1 - e^2})$$

$$\int_0^P \frac{dT_N}{r^3} = \frac{2\pi\mu}{h^3}$$

It follows that the first-order correction to the radial action for the observational metric is

$$\delta J_r = \frac{2\pi m \mu^2}{c^2 h} (3 - 2\sqrt{1 - e^2})$$

Using the Newtonian elliptical-orbit relation between energy and eccentricity, $1 - e^2 = -2\epsilon_N h^2/\mu^2$, together with $J = mh$, this becomes

$$\delta J_r = \frac{6\pi\mu^2 m^2}{c^2 J} - \frac{4\pi\mu m}{c^2} \sqrt{-2\epsilon_N}$$

The second term on the right-hand side depends only on the conserved energy and therefore vanishes when differentiated with respect to angular momentum at fixed energy. From the Hamilton–Jacobi relation, the azimuthal angle Φ_r accumulated during one complete radial period is

$$\Phi_r = - \left. \frac{\partial J_r}{\partial J} \right|_{\varepsilon}$$

For an unperturbed closed ellipse, Φ_r equals 2π . Accordingly, the perihelion advance produced by the perturbation is

$$\Delta\varpi = - \left. \frac{\partial(\delta J_r)}{\partial J} \right|_{\varepsilon} = \frac{6\pi\mu^2 m^2}{c^2 J^2}$$

$$\Delta\varpi = \frac{6\pi GM}{a(1-e^2)c^2}$$

This agrees with the perihelion-advance formula derived from the orbital equation in Section 16.2. Since the radial-action correction produces the phase difference $\Delta\theta_{m,r} = \delta J_r/\hbar$, the orbital advance can be expressed through the dependence of this phase difference on angular momentum.

The radial action evaluated in this section is a canonical quantity associated with the observational metric. By contrast, the angular momentum $h_T = r^2 d\phi/dT$ of a real orbit parametrized by universal time T does not generally equal the phase-characteristic angular momentum h . The next section constructs the correspondence under which the two descriptions give the same orbital shape, without assuming that their time parameters are identical in all respects.

17. Correspondence Between Real Orbits in Universal Time and Phase-Characteristic Curves

17.1 Strict Distinction Between Time Parameters

The constants arising from separation of the phase determine the time correspondence along a characteristic curve:

$$\frac{dT}{d\sigma} = \frac{\varepsilon}{f(r)}$$

Therefore,

$$\frac{d\phi}{dT} = \frac{f(r)h}{\varepsilon r^2}, \quad \frac{dr}{dT} = \frac{f(r)}{\varepsilon} \frac{dr}{d\sigma}.$$

In this representation, T is identified with the reference coordinate of universal time. The orbital shape $r(\phi)$ is determined by the phase-characteristic curve, but $h = J/m$ is the conserved quantity along that curve and is not generally equal to $r^2 d\phi/dT$ defined using universal time. Defining the correspondence between positions in universal space and the coordinates r, ϕ of the observational metric and adopting these time relations make it possible to construct matter orbits parameterized by universal time.

17.2 A Newtonian-Type Corrected Law of Motion Giving the Same Orbital Shape

Alternatively, independently construct a central-force model that conserves the angular momentum $h_T = r^2 d\phi/dT$ defined with universal time, and take

$$m \frac{d^2 \mathbf{X}}{dT^2} = - \frac{GMm}{r^2} \left(1 + \frac{3h_T^2}{c^2 r^2} \right) \hat{\mathbf{r}}$$

The work–energy integral for this central force is

$$E_T = \frac{m}{2} \left(\frac{dr}{dT} \right)^2 + \frac{mh_T^2}{2r^2} - \frac{GMm}{r} - \frac{GMmh_T^2}{c^2 r^3}.$$

Applying the same variable substitution $u = 1/r$ gives

$$\frac{d^2 u}{d\phi^2} + u = \frac{GM}{h_T^2} + \frac{3GM}{c^2} u^2$$

This Newtonian-type corrected law in real time gives an orbital shape $r(\phi)$ of the same form as that obtained from matter-wave phase transport. Nevertheless, the conserved h of the characteristic curve generally differs from h_T defined in real time; thus, coincidence of every time assignment does not follow automatically. An additional correspondence rule is required to identify the two parameterizations.

18. Formulas Corresponding to SR and GR and Their Unification Within NNRT

Combining the electromagnetic observational mapping of LT, the gravitational action of the observational-scale field, electromagnetic phase transport, matter-wave action phase, and clock-phase recording gives the following relations under a common mathematical normalization.

- Uniformly moving observation: $t'_{\text{obs}} = \gamma(T - vX/c^2)$; $x'_{\text{obs}} = \gamma(X - vT)$.
- Relativistic Doppler relation: $\nu'/\nu = \sqrt{(1 - \beta)/(1 + \beta)}$ (receding direction).
- Observed light speed: $|\omega'/k'| = c$ (a consequence of the observational mapping for the vacuum dispersion relation).
- Gravitational redshift: $\nu_r/\nu_e = \sqrt{f(r_e)/f(r_r)}$.
- Solar-limb light deflection: $\Delta\theta \simeq 4GM/(bc^2)$, approximately 1.75 arcseconds.
- Shapiro delay: logarithmic propagation delay proportional to $2GM/c^3$.
- GPS circular-orbit clock: $C_{\text{GPS}} = \sqrt{1 - 3GM/(rc^2)}$, with an approximate difference of 38.6 $\mu\text{s}/\text{day}$ relative to the terrestrial reference.
- Mercury's perihelion advance: $\Delta\varpi = 6\pi GM/[a(1 - e^2)c^2]$, approximately 42.98 arcseconds per century.

These mathematical agreements are obtained by explicitly specifying the field equations of the observational metric, the coupling laws of the wave fields, measurement boundary conditions, and physical constants.

19. Distinct Physical Roles of GT and LT and the NNRT Reconstruction of GR

Conventional SR uses flat four-dimensional Minkowski spacetime, whereas GR uses generally curved metric geometry; the observational relations of matter, clocks, and light are described through those geometric structures. NNRT reverses this direction of construction: universal time and universal space are the real foundation, and observational spacetime is constructed from real motion related by GT, electromagnetic and matter-wave phases, and the scales of measuring instruments.

GT expresses how universal spacetime is shared among inertial frames. LT, on the other hand, relates the two-way measurement of light, phase, frequency, wavenumber, and observational time–distance scales along the GT worldlines. They are therefore not two notations for the same physical object. The former concerns real motion, the latter electromagnetic observation.

Likewise, in NNRT's gravitational observational theory, gravitational sources and acceleration give rise to an observational coframe, on which the null phase of electromagnetic waves, action phase of matter waves, and accumulated cycle counts of atomic clocks are specified. The four-dimensional observational geometry appearing in the formulas for light, clocks, and particle orbits is understood as spacetime drawn by these measurement scales.

Phase invariance must be distinguished from changes in observed phase gradients. Although the phase θ is invariant for the same wavefront event, $\omega_{\text{obs}} = -d\theta/dt_{\text{obs}}$ and its spatial gradients may depend on the propagation environment and the chosen measurement scales. This distinction allows the empirical LT derivation, gravitational redshift, GPS, and Mercury's perihelion advance to be addressed in a single mathematical system.

20. Conclusion

This paper began by postulating universal time, universal space, and GT and then constructing a thought experiment based on emission, reflection, and reception of light recorded with the same universal clock. From the asymmetry of the actual outward and return times and the classical Doppler relations, we constructed an optically symmetric scale and derived LT by assigning times to arbitrary events through light emission and reception. The result was shown to coincide with the rigorous derivation obtained by normalizing the coefficients of the vacuum wave operator. Phase invariance then yielded the relativistic Doppler formula and invariance of observed light speed.

For gravitational fields, we introduced an observational-scale field on universal spacetime and derived the Schwarzschild observational metric, electromagnetic phase transport, gravitational redshift, light deflection, and Shapiro delay. For GPS, we emphasized that a satellite has centripetal acceleration rather than uniform inertial velocity and reproduced the frequency and time corrections by including both the

gravitational source and circular motion in the construction of the observational scale. We further introduced matter-wave phase transport and derived Mercury's perihelion advance both analytically and numerically from an effective potential containing the r^{-3} correction. In addition, the canonical radial action of the matter wave was integrated over one complete period, independently verifying the same perihelion advance from the first-order correction that includes the radial component of the observational metric.

Throughout this construction, universal time T remains unchanged. What changes are the observed phase gradients of electromagnetic and matter waves, optical propagation, clock oscillation scales, and the resulting measurement records. The four-dimensional observational spacetime constructed by NNRT is formulated as the spacetime depicted by these scales and measurement procedures.

The central feature of NNRT's reconstruction of relativity is to retain universal spacetime as the real foundation, construct LT and gravitational observational geometry from wave phase and observational scales along GT motion, and thereby preserve the corresponding relativistic observational formulas and numerical predictions.

Acknowledgments: This paper has been edited to supplement the explanations presented in references [2] and [3]; these publications should therefore be read together with the additional explanations supplied here. The manuscript was prepared with AI assistance in drafting, organizing, and systematizing the text following an extended theoretical dialogue with the author. Responsibility for the theoretical claims and physical interpretations rests with the author.

Appendix A. Definitions of Symbols and Variables

T : universal time; $X, \mathbf{X}$: coordinates of universal space; ξ : moving coordinates defined by GT; $v, \mathbf{v}$: relative velocity between inertial frames; c : speed of light in vacuum; $\beta = v/c$: dimensionless velocity; $\gamma = (1 - \beta^2)^{-1/2}$: coefficients of the observational mapping; L : GT-defined rod length; T_+, T_- : actual universal times for outward and return light travel; T_b : arithmetic mean of those times; T_{sym} : optically symmetric time obtained from the geometric mean; $t'_{\text{obs}}, x'_{\text{obs}}$: electromagnetic observational coordinates; θ : electromagnetic wave phase; ω, ν, k : angular frequency, frequency, and wavenumber; G : gravitational constant; M : mass of the gravitational source; m : mass of the test particle; r, ϕ : spherically symmetric coordinates of the observational metric; e_μ^a : observational coframe; $g_{\mu\nu}^{\text{obs}}$: observational metric; τ_{obs} : observational time scale; C_g, C_{orb} : observational-scale factors of stationary and circular-orbit clocks; λ : parameter along a light-phase characteristic; σ : parameter along a matter-wave phase characteristic; $\mathcal{E}, J$: conserved phase energy and angular momentum of matter waves; $h = J/m$: specific angular momentum of the phase-characteristic curve; $h_T = r^2 d\phi/dT$: specific angular momentum in the universal-time central-force model; a : orbital semimajor axis (distinguished contextually from a coframe index); e : orbital eccentricity (distinguished contextually from the coframe determinant); N : number of recorded clock oscillation cycles; $\mu = GM$ is the gravitational parameter; ϵ_N is the orbital energy per unit mass in the weak-field limit; Q_0 is the squared radial velocity on the reference Newtonian orbit; p_r is the canonical radial momentum obtained from the observational phase; J_r is the radial action over one complete period; δJ_r is its first-order correction; J is the phase angular momentum; Φ_r is the azimuthal angle accumulated over one radial period; and T_N is the time parameter of the zeroth-order Newtonian orbit used in evaluating the radial action.

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